9.2.2.2 Represent and solve problems in various contexts using exponential functions, such as investment growth, depreciation and population growth

9.2.4.2 Represent relationships in various contexts using equations involving exponential functions; solve these equations graphically or numerically.

Application Preview Pg. 259-260

Chapter 4 Exponential and Logarithmic Functions

Lesson 4.1 Exponential Functions

Notes

Exponential Functions

Exponential functions are functions that have variables in the exponent.

Exponent
$f(x) = a^x$ $a > 0$ $, a \neq 1$
Base

Exponential functions with bases $a > 1$ are used to model growth.
Exponential functions with bases $0 < a < 1$ are used to model decay.

The domain for an exponential function will be the set of Real numbers and the range will be the set of real numbers greater than 0.

Compound Interest
For P dollars invested at annual interest rate r,

$A = Pe^{rt}$

Value after n periods = $P(1 + r/n)^{nt}$
P = principal
r = annual rate of return
n = number of compoundings
t = time

Example

Find the value of $2000 invested for 2 years at an annual rate of 36% compounded monthly.

Present Value

For P dollars, at annual interest rate r for n periods,
Present value = $P/(1 + r/n)^{nt}$

Depreciation by a fixed percentage

Depreciation by a fixed percentage means that a piece of equipment loses a fixed percentage of its value each year. To determine the value we will use the interest formula and simply use a negative rate to represent the loss of value.

Example

A printing press, originally worth $50,000, loses 20% of its value each year. What is its value after 4 years?

Assignment Pg. 271; 13-15, 19-20, 25-26

Notes

The constant e

$e = \lim_{n \to \infty} (1 + 1/n)^n = 2.71828...$

Continuous Compounding

For P dollars invested at interest rate r compounded continuously,

$V = Pe^{rt}$

Present Value with Continuous Compounding

For P dollars and interst rate r compounded continuously for n years,

Present value = $P/e^{rt} = Pe^{-rt}$

Examples 4 & 5 & 6 Pg. 267-268

APR

Annual Percentage Rate represents the actual increase in one year. The nominal rate of interest is the stated interest that you will receive. The nominal rate and the APR aren't always the same.

The Function $y = e^x$

Assignment Pg. 271-275; 1-20, 23-26, 28-44/4

Lesson 4.2 Logarithmic Functions

Notes

Logarithms

A logarithm is an exponent

Logarithms to the base 10 are called common logarithms.

Example

Find $\log_{10} 10{,}000$

Notes

In general $\log_a x = y$ is equivalent to $a^y = x$

Example

Find

$\log_5 125$

$\log_8 2$

Notes

Natural logarithm

Logarithms to the base e are called natural logarithms.

$\log_e = \ln$

Example

Find ln 8.34

Notes

Properties of natural logarithms

1) $\ln 1 = 0$
2) $\ln e = 1$
3) $\ln e^x = x$
4) $e^{\ln x} = x$
5) $\ln (M*N) = \ln M + \ln N$
6) $\ln (1/N) = - \ln N$
7) $\ln (M/N) = \ln M - \ln N$
8) $\ln (M^N) = N \ln M$

Assignment Pg. 287-291; 1-16, 18-51/3

Compound Interest

Example

A sum of money is invested at 10% interest compounded semiannually (twice a year). How soon will it increase by 60%.

Drug Dosage

If the initial concentration is c (milligrams per milliliter of blood), the concentration t hours later will be

$C(t) = ce^{-kt}$

where the absorption constant k measures how rapidly the drug is absorbed.

Example 11 Pg. 283

http://books.google.com/books?vid=ISBN0852007124&id=HUV4FmpDV4MC&pg=RA1-PA97&lpg=RA1-PA97&ots=mrRsZnjT6B&dq=Mathematical+determination+of+Drug+Dosage&sig=_DKb88Ii1kOmvUC4SsMNx1A45q8#PPR2,M1

Carbon 14 Dating

Proportion of carbon 14 remaining after t years = $e^{-.00012t}$

Example 12 Pg. 284

Behavioral Science: Learning Theory

Your skill after t units of practice is given by a function of the form,

$S(t) = c(1 - e^{-kt})$

where c and k are positive constants.

Example 13 Pg. 285

Social Science: Diffusion of Information by Mass Media

When a news bulletin is repeatedly broadcast over the radio and television, the proportion of people who hear the bulletin within t hours is

$p(t) = 1 - e^{-kt}$

for some constant k.

Assignment Pg. 287-291; 1-16, 18-51/3

Lesson 4.3 Differentiation of Logarithmic and Exponential Functions

Notes

Derivatives of Logarithmic Functions

Derivative of ln x

d/dx ln x = 1/x Pg. 302 for verification

$f(x) = \frac{\ln x}{x}$

$f'(x) = \frac{x\left(\frac{1}{x}\right) - \ln x(1)}{}$

Example 1 Pg. 292

Practice

Differentiate $f(x) = \frac{\ln x}{x}$

Derivative of ln f(x) (f(x) must be positive)

d/dx ln f(x) = f'(x)/f(x) Pg. 302 for verification

Example 2 Pg. 293

$\frac{d}{dx} \ln (x^4-1)^3 = \frac{1}{} \cdot 3(x^4-1)^2(4x^3)$

Practice

Find d/dx ln $(x^3 - 5x + 1)$

Example 3 Pg. 293

$f(x) = x e^x$

Derivatives of Exponential Functions

Derivative of e^x

d/dx $e^x = e^x$ Pg. 302-303 for verification

Example 4 and 5 Pg. 294-295

$\frac{d}{dx} e^{1+\frac{x^3}{3}}$

Practice

If $f(x) = xe^x$, find f'(1)

Derivative of $e^{f(x)}$

d/dx $e^{f(x)} = e^{f(x)} * f'(x)$

Example 6 and 7 Pg. 295

Practice

Find d/dx $e^{1+x3/3}$

Example 8 Pg. 297

Derivative of e^{kx}

d/dx $e^{kx} = ke^{kx}$

Assignment Pg. 303-307; 1-61odd

Lesson 4.3 Cont.

Notes

Maximizing Consumer Expenditure

Let D(p) be the consumer demand at price p. Then consumer expenditure is
$E(p) = p * D(p)$

Example

Consumer demand for a commodity can be modeled by the following function.

Find the price that maximizes consumer expenditure.

$D(p) = 5000e^{-.01p}$

Graphing Logarithmic and Exponential Functions

We graph logarithmic and exponential functions using the same techniques as before.

Example

Graph the following functions

$f(x) = e^{-2x2}$

$f(x) = \ln (1 + x^2)$

Assignment Pg. 304-305 75-90/5

Lesson 4.3 Cont.

Notes

Derivatives of a^x and $a^{f(x)}$

$d/dx\ a^x = (\ln a)a^x$

$d/dx\ a^{f(x)} = (\ln a)a^{f(x)}f'(x)$

Example

Find the derivative of the following

$f(x) = 10^x$ $\quad f'(x) = \ln 10(10^x)$

$f(x) = 3^{x^2 + 1}$ $\quad f'(x) = (\ln 3)(3^{x^2+1})(2x)$

Notes

Derivatives of $\log_a x$ and $\log_a f(x)$

$d/dx\ \log_a x = 1/(\ln a)x$

$d/dx\ \log_a f(x) = f'(x)/(\ln a)f(x)$

$\log_2 x \quad \frac{1}{(\ln 2)x}$

Example

Find the derivative of the following

$\log_2 x$

$\log_{10}(x^2 + 1)$ $\quad f(x)$

$$\frac{2x}{(\ln 10)(x^2+1)}$$

Assignment Pg. 306-308; 95-115; odd

Application of exponentials and logarithms

February 13, 2019