January 2, 2019

Application Preview Pg. 179

Chapter 3 Further Applications of Derivatives

Lesson 3.1 Graphing using the first derivative

Notes:

Calculus enables us to find the most important points on a curve.

Since the derivative of a function gives us the slope of the graph, if $f' > 0$ on an interval, then f is increasing on that interval. Conversely, if $f' < 0$ on an interval, then f is decreasing on that interval.

Relative Extreme Points and Critical Numbers:

A relative maximum point is a point that is at least as high as the neighboring points of the curve on either side, and a relative minimum is a point that is at least as low as the neighboring points on either side.

Notation wise it means that f has a relative maximum value at c if $f(c) \geq f(x)$ for all values of x near c. Likewise for a relative minimum except $f(c) \leq f(x)$ for all values of x near c.

A **critical number** of a function f is an x-value in the domain of f at which either

$f'(x) = 0$ or $f'(x)$ is undefined.

Graphing Functions

We graph a function by finding its critical numbers, making a "sign diagram" for the derivative to show intervals of increase and decrease and the relative extreme points, and then drawing the curve.

Example 1 Pg. 182

Assignment Pg. 191-194; 3-63/3

Notes Cont.

First-Derivative Test

If a function has a critical number c, then at $x = c$ the function has a relative maximum if $f' > 0$ just before c and $f' < 0$ just after c. A relative minimum if $f' < 0$ just before c and $f' > 0$ just after c.

Example 2 Pg. 185

Notes Cont.

Graphing Rational Functions

Vertical Asymptote

A rational function $p(x)/q(x)$ has a vertical asymptote $x = c$ if

$q(c) = 0$ but $p(c) \neq 0$

Horizontal Asymptotes

A function $f(x)$ has a horizontal asymptote $y = c$ if

$\lim_{x \to \infty} f(x) = c$ or $\lim_{x \to -\infty} f(x) = c$

Example 3 Pg. 187

Example 4 Pg. 188

Practice

Explain why, for a rational function, the function and its derivative will be undefined at the same x-values. Are such x-values critical numbers?

Practice Problem 2 Pg. 189

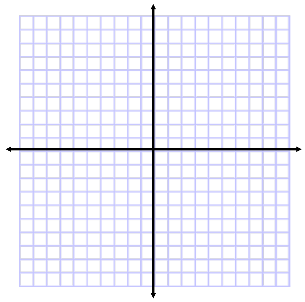
Assignment Pg. 191-194; 3-63/3, 64-84/4

January 2, 2019

Lesson 3.2 Graphing Using the First and Second Derivatives

Concavity and Inflection Points

A curve that curls up is *concave up*, and a curve that curls down is *concave down*. The point where the concavity changes is called an *inflection point.*



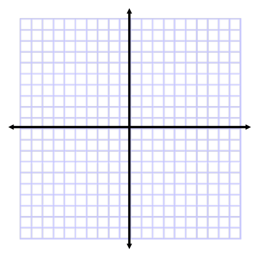
Practice problem Pg. 196

Concavity and Inflection Points

f'' > 0 on an interval means that f is *concave up* on that interval.

f'' < 0 on an interval means that f is *concave down* on that interval.

An inflection point is where the concavity changes (f'' must be zero or undefined).



Example 1 Pg. 197

Practice Problem 2 Pg. 199

Notes Cont.

Inflection Points in the Real World

Example 2 Pg. 200

Example 3 Pg. 201

Example 4 Pg. 202

Second Derivative Test for Relative Extreme Points

If x = c is a critical number of f at which f'' is defined, then f''(c) > 0 means that f has a relative *minimum* at x = c. f''(c) < 0 means that f has a relative *maximum* at x = c.

Note: If the second derivative is equal to 0 we must investigate further to determine if the point is a minimum or a maximum.

Curve Sketching Summary

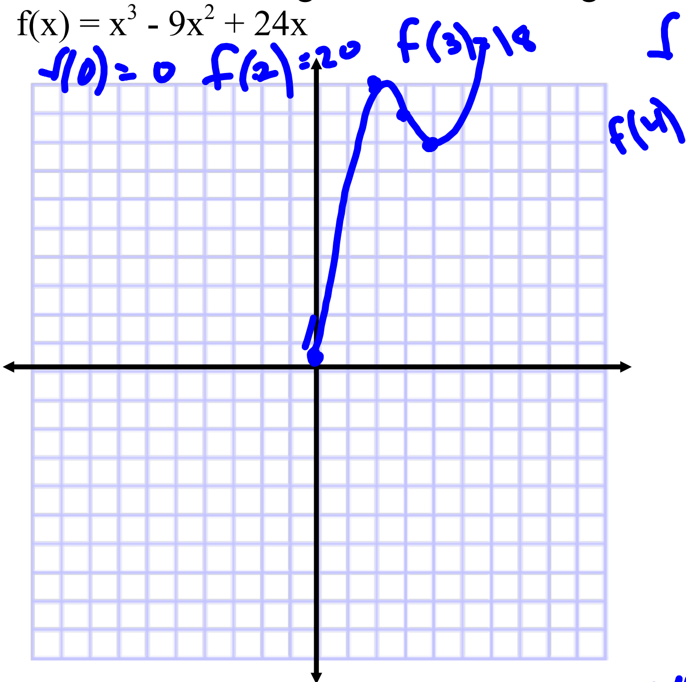
1) Find the domain of the function.
2) Find the first derivative.
3) Set the derivative equal to zero and solve for x. Also, identify where the derivative is undefined.
4) Find the corresponding y value for each x value in the domain of the function.
5) Create a sign diagram.
6) Find the second derivative.
7) Set the second derivative equal to zero and solve for x. Also, find the x values where the second derivative is undefined.
8) Find the corresponding y value for each x in the domain of the function.
9) Make a sign diagram for the second derivative marking the inflection points.
10) From the sign diagrams construct the graph.

Assignment Pg. 205-209; 3-45/3, 60-85/5

Practice

Sketch the following function indicating the CN and Inflection points.

$f(x) = x^3 - 9x^2 + 24x$

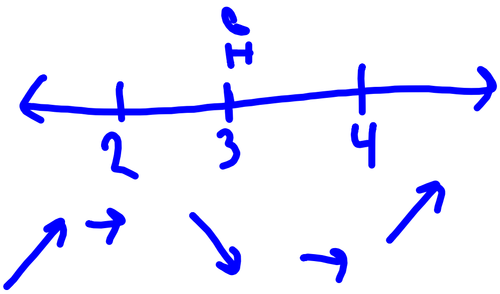


$f'(x) = 3x^2 - 18x + 24 = 0$

$3(x^2 - 6x + 8) = 0$

$3(x-4)(x-2) = 0$

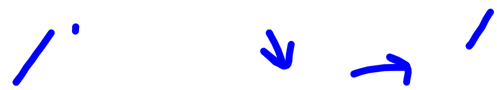
CN @ $x = 4$ and $x = 2$



$f''(x) = 6x - 18 = 0$

$x = 3$

Assignment Pg. 211-213; 1-25o



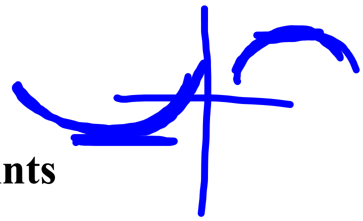
January 2, 2019

Inflection Points in Everyday Life

Example 1 Pg. 202

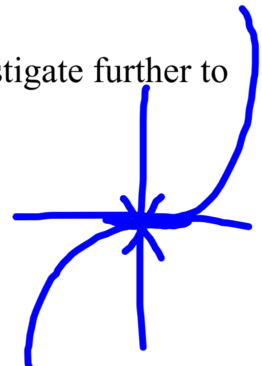
Page 204-205.

$f'(x) = 0$



Second Derivative Test for Relative Extreme Points

If x = c is a critical number of f at which f" is defined, then
f"(c) > 0 means that f has a relative *minimum* at x = c.
f"(c) < 0 means that f has a relative *maximum* at x = c.
Note: If the second derivative is equal to 0 we must investigate further to determine if the point is a minimum or a maximum.



Curve Sketching Summary

1) Find the domain of the function.
2) Find the first derivative.
3) Set the derivative equal to zero and solve for x. Also, identify where the derivative is undefined.
4) Find the corresponding y value for each x value in the domain of the function.
5) Create a sign diagram.
6) Find the second derivative.
7) Set the second derivative equal to zero and solve for x. Also, find the x values where the second derivative is undefined.
8) Find the corresponding y value for each x in the domain of the function.
9) Make a sign diagram for the second derivative marking the inflection points.
10) From the sign diagrams construct the graph.

Assignment Pg. 211-213; 1-49o, 53-63o

Lesson 3.3 Optimization

Notes

Absolute Extremes

The absolute maximum value of a function is the largest value of the function on its domain and similarly for the absolute minimum.

Optimizing Continuous Function on Closed Intervals

A continuous function f on a closed interval [a,b] has both an absolute maximum value and an absolute minimum value. To find them:

1. Find all critical numbers of f in [a,b].
2. Evaluate f at the critical numbers and at the endpoints a and b.
3. The largest and smallest values found in step 2 will be the absolute max. and min. of f on [a,b].

Example 1 Pg. 211
Practice

Find the absolute extreme values of each function on the given interval.
$f(x) = x^3 - 6x^2 + 9x + 8$ on [-1,2]

Applications of Optimization

Optimizing the value of a timber forest

Profit, Revenue, Cost

Area

Volume

Example #2 Pg. 212

Notes

Maximizing Profit

Profit = Revenue - Cost

Revenue = (unit price) Quantity

Price Function

p(x) gives the price at which consumers will buy exactly x units of the product.

Example 3 Pg. 214

Maximum Profit

At maximum profit (Marginal Revenue) = (Marginal Cost)

Example #4 Pg. 216

Example # 5 Pg. 217

Assignment Pg. 219-223; 1-23 odd, 25-65/5

Lesson 3.4 Further Applications of Optimization

Notes

In this section we will continue to focus on optimization but we will focus specifically on problems where we are given information that describes how price changes will affect sales.

Revenue = Price/item * Quantity
Cost = Cost/item * Quantity
Profit = Revenue - Cost
Government Revenue = tax rate * total sales

Example 1 Pg. 224

Practice

A computer manufacturer can sell 1500 personal computers per month at a price of $3000 each. The manager estimates that for each $200 price reduction he will sell 300 more each month. If x stands for the number of $200 price reductions, express the price p and the quantity q as functions of x. If each computer costs the manufacturer $1000 to build, find the price and the number of computers that the manufacturer should build and sell to maximize his profit.

Example 2 Pg. 225
Example 3 Pg. 226
Example 4 Pg. 227
Example 5 Pg. 229

Assignment Pg. 230-232; 1-35 odd

Lesson 3.5 Optimizing Lot Size and Harvest Size

Notes

Minimizing Inventory Cost

Example 1 Pg. 233

Production Runs

Example 2 Pg. 235

Maximum Sustainable Yield

Reproduction Function

A reproduction function f(p) gives the population a year from now if the current population is p.

$Y'(p) = f'(p) - 1$

$= 0$

$f'(p) = 1$

Sustainable Yield

For reproduction function f(p), the sustainable yield is $Y(p) = f(p) - p$

Maximum sustainable yield

For reproduction function f(p), the population p that results in the maximum sustainable yield is the solution to

$f'(p) = 1$

The maximum sustainable yield is then

$Y(p) = f(p) - p$

Example 3 Pg. 238

Assignment Pg. 239-240; 1-27 odd

Lesson 3.6 Implicit Differentiation and Related Rates

Notes

A function written in the form y = f(x) is said to be defined explicitly, meaning that y is defined by a rule or formula in x alone. A function that is defined implicitly is one in which the variable y is difficult or impossible to express in terms of x.

Example 1 Pg. 241
Example 2 Pg. 242
Example 3 Pg. 243

Practice

Find

$d/dx\ x^4$
$d/dx\ y^2$
$d/dx\ (x^2y^3)$

Example 4 Pg. 243

Assignment Pg. 249; 1-35 odd

The following website provides an effective tool to help understanding http://archives.math.utk.edu/visual.calculus/3/implicit.7/index.html

Notes

A demand equation is the relationship between the price *p* and the quantity *x* that consumers w demand at that price.

Example 5 Pg. 244

Practice

A company's demand equation is $x = \sqrt{68 - p^2}$, where p is the price in dollars. Find dp/dx when p = 2 and interpret your answer.

Notes Cont.
Related Rates

We refer to problems where both variables in an equation are functions of a third, generally time, as related rate problems. Solving related rate problems requires us to differentiate with respect to that third variable.

Example 6 Pg. 245

Practice

A large snowball is melting so that its radius is decreasing at the rate of 2 inches per hour. How fast is the volume decreasing at the moment when the radius is 3 inches?

Example 7 Pg. 246
Example 8 Pg. 247

Assignment Pg. 249-252; 1-39 odd, 42-81/3