Get Ready Pg. 277; 1-17 odd

Standards
9.2.2.6
9.2.3.1

Chapter 5 Polynomials and Polynomial Functions
Lesson 5-1 Polynomial Functions

Objectives
1) Classify polynomials
2) Graph polynomial functions and describe end behavior

Vocabulary
Monomial
Degree of a monomial
Polynomial
Degree of a polynomial
Polynomial function
Standard form of a polynomial function
Turning point
End behavior

Notes
A ***monomial*** is a real number, a variable, or a product of a real number and one or more variables with whole-number exponents.
The ***degree of a monomial*** in one variable is the exponent of the variable.
A ***polynomial*** is a monomial or the sum of monomials.
The ***degree of a polynomial*** is the largest degree of any term in the polynomial.
The ***degree of a term*** is determined by the exponent of the variable in that term.

Definition-Standard Form of a Polynomial Function
$P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$ where n is a nonnegative integer and $a_n, \ldots, a_0$ are real numbers
and the coefficient $a_n, \ldots,$
We can classify polynomials by degree or by number of terms.

Example
Write each polynomial in standard form. Then classify it by degree and by number of terms.

$9 + x^3$ — $x^3 + 9$ binomial
cubic
$x^3 - 2x^2 - 3x^4$ — $-3x^4 + x^3 - 2x^2$ trinomial

You can determine the end behavior of the graph of a polynomial function of degree n from the leading term ax^n of the standard form.

	n even	n odd
a positive	Up and Up	Down and Up
a negative	Down and Down	Up and Down

Example
Consider the leading term of $y = 3x^4 - 2x^3 + x - 1$. What is the end behavior of the graph?

In general, the graph of a polynomial function of degree n has at most n - 1 ($n \geq 1$) turning points. If n is odd, there will be an even number of turning points. Likewise, if n is even there will be an odd number of turning points.

$y = -2x^3 + x + 3$

Example
What is the graph of $y = 3 - 2x^3 + x$? Describe the graph.

What is the degree of the polynomial function that generates the data shown in the table?

x	y
-2	-13
-1	-4
0	-1
1	2
2	11
3	32
4	71

Assignment Pg. 285-287; 9-39 odd, 42-48/3

9.2.3.3 Factor common monomial factors from polynomials.

Lesson 5-2 Polynomials, Linear Factors, and Zeros

Objectives

1) Analyze the factored form of a polynomial
2) Write a polynomial function from its zeros

Vocabulary

Factor Theorem
Multiple zero
Multiplicity
Relative maximum
Relative minimum

Example

What is the factored form of $x^3 + x^2 - 12x$?

Notes

If P(x) is a polynomial function, the solutions of the related polynomial equation P(x) = 0 are the zeros of the function.

Factor Theorem

The expression x - a is a factor of a polynomial if and only if (iff) the value a is a zero of the related polynomial function.

$(x+4) = (x - (-4))$

Example

What are the zeros of $y = (x - 3)(x + 4)(x - 1)$? Graph the function.

What is a cubic polynomial function in standard form with zeros 1, -1, and 4?

$(x-1)(x+1)(x-4) = 0$

Find the zeros of $y = (x + 1)(x - 1)(x + 3)$.

Write a polynomial function in standard form with zeros at 2, -3, and 0.

Notes

A repeated zero is called a ***multiple zero*** and has a ***multiplicity*** equal to the number of times the zero occurs.

How Multiple Zeros Affect a Graph

If a is a zero of multiplicity n in the polynomial function y = P(x), then the behavior of the graph at the x-intercept a will be close to linear if n = 1, close to quadratic if n = 2, close to cubic if n = 3, and so on.

Example

Find any multiple zeros of $f(x) = x^5 - 6x^4 + 9x^3$ and state the multiplicity.

$x^3(x^2 - 6x + 9) = x^3(x-3)(x-3)$

0 mult of 3 3 mult of 2

What are the zeros of $f(x) = x^3 - 5x^2 + 3x + 9$? What are the multiplicities? How does the graph behave at these zeros?

Notes

The maximum value within a given interval of points is called a ***relative maximum***.
The minimum value within a given interval of points is called a ***relative minimum***.

Example

What are the relative maximum and minimum of $f(x) = x^3 - 9x$?

An airline has the following carry-on luggage regulations. The sum of the length, width, and depth may not exceed 50 in.

Suppose that the sum of the length, width, and depth is 50 in. and the length is 10 in. greater than the depth. Graph the function relating the volume V to the depth x. Find the x-intercepts. What do they represent?

Describe a realistic domain for V(x).

What is the maximum possible volume of the box? What are the corresponding dimensions of the box?

Assignment Pg. 293-295; 7-39 odd, 40-54

9.2.2.1 Represent and solve problems in various contexts using linear and quadratic functions

Lesson 5-3 Solving Polynomial Equations

Objectives

1) Solve polynomial equations by graphing.
2) Solve polynomial equations by factoring.

Vocabulary

Sum of cubes
Difference of cubes

Polynomial Factoring Techniques

Factoring the GCF
Quadratic Trinomials
Perfect Square Trinomials

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Difference of Squares

$a^2 - b^2 = (a + b)(a - b)$

Factoring by Grouping

$ax + ay + bx + by = a(x + y) + b(x + y) = (a + b)(x + y)$

Sum and Difference of Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a^2 - b^2$

$(a+b)(a-b)$

Example

Factor $x^3 - 125$

Solve $8x^3 + 125 = 0$

Factor $x^4 - 6x^2 - 27$

Solve $x^4 - 4x^2 - 45 = 0$

What are the real or imaginary solutions of $4x^3 - 6x^2 = 4x$?

What are the real or imaginary solutions of the equation $2x^3 = -54$

What are the real solutions of $2x^3 + 5 = 3x^2 - 2x$

What are three consecutive even integers whose product is 4 times their sum?

Example

Graph and solve $x^3 - 19x = -2x^2 + 20$.

Assignment Pg. 300-302; 11-39 odd, 42, 48, 51-53, 57

9.2.3.2 Add, subtract, and multiply polynomials; divide a polynomial by a polynomial of equal or lower degree.

Lesson 5-4 Dividing Polynomials

Objectives

1) Divide polynomials using long division.
2) Divide polynomials using synthetic division.

Vocabulary

Synthetic Division
Remainder Theorem

Notes

Division Algorithm for Polynomials

You can divide polynomial P(x) by polynomial D(x) to get polynomial quotient Q(x) and polynomial remainder R(x). The result is P(x) = D(x)Q(x) + R(x)

If R(x) = 0, then P(x) = D(x)Q(x) and D(x) and Q(x) are factors of P(x).

Examples

Divide the following

56 by 8

42 by 5

Use polynomial long division to divide:

$5x^2 + 2x + 3$ by $x + 1$

$x^2 + 2x - 30$ by $x - 5$

$x - 2$

Is $x^2 - 2$ a factor of $P(x) = x^4 - x^2 - 2$?

$x - 4$

$x + 2 : x - (-2)$

Determine whether x + 2 is a factor of each polynomial

$x^2 + 10x + 16$ $(x+2)(x+8)$

$x^3 + 7x^2 - 5x - 6$

Assignment Pg. 308-310; 9-39 odd, 40-43, 67-85

Notes

Synthetic division simplifies the process for dividing by a linear expression x - a. The process enables us to remove all the variable from the problem and work with just the coefficients and constants.

Example

Use synthetic division to divide. What are the quotient and remainder.

$5x^3 - 6x^2 + 4x - 1$ by $x - 3$

$$\begin{array}{r|rrrr} 3 & 5 & -6 & 4 & -1 \\ & \downarrow & 15 & 27 & 93 \\ \hline & 5 & 9 & 31 & 92 \end{array}$$

$5x^2 + 9x + 31$

$r\ 92$

$4x^3 - 3x^2 + 2x - 3$ by $x - 1$

The polynomial $x^3 + 9x^2 + 23x + 15$ expresses the volume, in cubic inches, of a box and the length is (x + 5) inches. What are the other two dimensions of the box?

Notes

Theorem-Remainder Theorem

If a polynomial P(x) of degree $n \geq 1$ is divided by (x - a), where a is a constant, then the remainder is P(a).

Example

Use synthetic division to find P(3) for $P(x) = x^4 - 2x^3 + x - 9$.

The volume in cubic feet of a shipping carton is $V(x) = x^3 - 6x^2 + 3x + 10$. The height of the carton is x - 5 feet.

a. Find linear expressions for the other dimensions. Assume that the length is greater than the width.

b. If the width of the carton is 4 feet, what are the other two dimensions?

Assignment Pg. 308-310; 9-39 odd, 40-43, 67-85

Lesson 5-6 The Fundamental Theorem of Algebra

Objective

Use the Fundamental Theorem of Algebra to solve polynomial equations with complex solutions.

Vocabulary

Fundamental Theorem of Algebra

Notes

Rational Root Theorem

Let $P(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ be a polynomial with integer coefficients. There are a limited number of possible roots of $P(x) = 0$.

Integer roots must be factors of a_0.

Rational roots must have reduced form p/q where p is an integer factor of a_0 and q is an integer factor of a_n.

Conjugate Root Theorem

If P(x) is a polynomial with rational coefficients, then irrational roots of $P(x) = 0$ that have the form $a + \sqrt{b}$ occur in conjugate pairs. That is, if $a + \sqrt{b}$ is an irrational root with a and b rational, then $a - \sqrt{b}$ is also a root.

If P(x) is a polynomial with real coefficients, then the complex roots of

$P(x) = 0$ occur in conjugate pairs. That is, if a + bi is an irrational root with a and b real, then a - bi is also a root.

The Fundamental Theorem of Algebra

If P(x) is a polynomial of degree $n \geq 1$, then $P(x) = 0$ has exactly n roots,

including multiple and complex roots.

Example

What are all the complex roots of $x^5 - x^4 - 7x^3 + 7x^2 - 18x + 18 = 0$.

What are the zeros of $f(x) = x^4 + 2x^3 - 4x^2 - 7x - 2$?

Assignment Pg. 322-324; 9-29 odd, 38-46

Lesson 5-8 Polynomial Models in the Real World

Objectives

1) Fit data to linear, quadratic, cubic and quartic models.

Vocabulary

Notes

The (n + 1) Point Principle

For any n + 1 points in the coordinate plane that pass the vertical line test, there is a unique polynomial of degree at most n that fits the points perfectly.

Example

What polynomial function has a graph that passes through the points (-2,13),(0,9),(1,4) and (2,5)?

What is the best linear/quadratic model that best fits the data? Use the best fit model to estimate egg consumption in 1995.

Year	US Per Capita Egg Consumption
1970	302
1980	266
1990	231
2000	247

Assignment Pg. 335-337; 9-35 odd, 38-42

9.2.1.9 Determine how translations affect the symbolic and graphical forms of a function. Know how to use graphing technology to examine translations.

9.2.2.6 Sketch the graphs of common nonlinear functions and translations of these functions. Know how to use graphing technology to graph these functions.

Lesson 5-9 Transforming Polynomial Functions

Objective

Apply transformations to graphs of polynomials

Vocabulary

Power Function
Constant of Proportionality

Notes

In general, $y = a(x - h)^3 + k$ represents all of the cubic functions you can obtain by stretching, compressing, reflecting, or translating the cubic parent function $y = x^3$.

Example

What cubic function do you obtain by applying the following transformations to $y = x^3$: vertical stretch by the factor 3; reflection in the x-axis; vertical translation 5 units up?

What are the real zeros of $y = 1/2(x - 2)^3 - 3$?

Notes

Power Functions

A power function is a function of the form $y = a * x^b$, where a and b are nonzero real numbers.
The constant a is the constant of proportionality.

Examples

What is a quartic function with only two real zeros: $x = -4$ and $x = -6$?

Assignment Pg. 343-345; 7-25 odd, 35-41 odd, 47-58

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